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Numerical modelling of impacts on granular materials with a combined discrete – continuum approach

A. Breugnot
EGIS Géotechnique, Seyssins, France
L3S-R, UJF-INPG-CNRS, Grenoble Universités, Grenoble, France

Ph. Gotteland & P. Villard
L3S-R, UJF-INPG-CNRS UMR 5521, Grenoble Universités, Grenoble, France

ABSTRACT: This paper investigates modelling of granular material submitted to high energy impact due to block impact. An original combined discrete – continuum method is proposed which permits to use discrete element method to model precisely the complex behaviour of granular material in the vicinity of the impacted zone while a continuum approach is used in farther areas. Coupled methods proposed are validated through simple static and dynamic tests and, finally used to simulate high energy impact of a cubic impactant on a gravel layer.

1 INTRODUCTION

The anthropization of mountainous regions raises the problem of infrastructures (roads, railways, industrial areas ...) and inhabitants' protection against risk of rockfall. Zones defined as potentially risky, can be protected with civil engineering structures placed upward to stop or deviate the trajectory of a falling rock. Among structures protections, the dams (embankments) and cushions covering rock sheds, benefit from the capacity of geomaterials to absorb energy and distribute loading through the structure.

The behaviour of ground structures submitted to local impact due to rockfall is quite complex to be characterized precisely. Indeed, high energy loading induces large and irreversible deformations, high strain rates and stresses in geomaterials which make difficult the prediction of mechanical behaviour of rockfall protection structures.

Consequently, their design suffers from a lack of regulation and is often limited to static stability consideration, and only a few approaches (Tissières 1999, Ronco 2009) estimate the dynamical component of impact braking force.

At this time the most sophisticated numerical codes may assist in the analysis of the dynamical phenomena induced by impact loading, using either continuum (Pichler et al. 2008, Peila et al. 2007) or discrete approaches (Plassiard 2007, Calvetti et al. 2005, Bertrand et al. 2005). Indeed, models are often calibrated with experimental results, and permit to enlarge analysis to other configurations tests (parametric study).

Considering the granular nature of geomaterials, and numerous rearrangements and fractures that take place in the most solicited areas, the Distinct Element Method (DEM) (Cundall & Strack 1979) seems to be particularly adapted to model local mechanical behaviours in geomaterials under dynamical impact. However, the use of this approach, to model large scale works, requires a large number of particles, which increases both the times of modelling and computation.

In order to improve the design of larges structures, modelling needs to maintain accuracy in highly stressed areas while representing the mechanical behaviour of the global structure.

Consequently, an innovative and original multiscale approach is developed (Xiao & Belytschko 2004) to improve the computational efficiency: a discrete element method is coupled with a continuum mechanical model, which resolution can be far coarser than in DEM. Thereby, continuum domain, adapted to materials with non significant discontinuities, is used as a boundary condition.

The first part of the paper is devoted to the procedure of the combined discrete – continuum elements method. The use of such combined approach for granular material is validated through a simple configuration tests, in static and dynamic applications. In the second part, simulations, modelling an impact with a cubic impactant on a soil layer, are led to show the prospects of this combined approach.

2 COMBINED DISCRETE – CONTINUUM METHOD

2.1 *General coupling method and methodology*

Research concerning combined discrete – continuum approaches began in the early eighties, and was led in physical domain in order to study material's

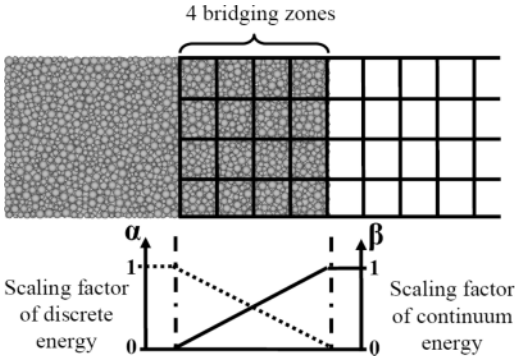

Figure 1. Discrete and continuum scaling factors in overlapping domain.

behaviour at molecular scale. Since then, the coupling methods and domains application were widely developed (Munjiza 2004, Itasca 2006, Onate 2003, ...). Xiao & Belytschko (2004) and Xiao & Hou (2007) proposed two different decompositions of domains which permitted to link continuum and discrete domains, either with an "edge-to-edge" method or with a "bridging" domain method, to study dynamical wave propagation or crack propagation in micromechanical structures.

In the latter case, discrete and continuum domains are overlapped in a bridging subdomain, where Hamiltonian H is taken to be a linear combination of the discrete and continuum total energies (Fig. 1), respectively $H_{Discrete}$ and $H_{Continuum}$ (Equation 1).

$$H = \alpha \ H_{Discrete} + \beta \ H_{Continuum} \tag{1}$$

In bridging zone, discrete and continuous displacements are linked to ensure continuity between the two domains. The displacement $\mathbf{d_j(X_j)}$ of a discrete particle j, localized by vector position $\mathbf{X_j}$, is written $\mathbf{d_j}$ (Equation 2). The continuum displacement $\mathbf{u(X_j)}$, at the same localization $\mathbf{X_j}$, can be expressed in terms of displacement $\mathbf{u_i}$ of the 8 nodes $i = a$ to g, which surround discrete particle j, by the mean of kinematic relations $\mathbf{k_{ji}}$ (Equation 3). The two domains are finally constrained via Equation 4: discrete displacements are required to conform to the continuum displacements at the positions of particles. The difference between discrete and continuum displacements is characterized by vector of residual displacements $\mathbf{g}$.

$$\overrightarrow{d_j(\overrightarrow{X_j})} = \overrightarrow{d_j} \tag{2}$$

$$\overrightarrow{u(\overrightarrow{X_j})} = \sum_{i=a}^{g} \overline{\overline{k_{ji}}} \ \overrightarrow{u_i} \tag{3}$$

$$\vec{g} = \overrightarrow{d_j(\overrightarrow{X_j})} - \overrightarrow{u(\overrightarrow{X_j})} = \vec{0} \tag{4}$$

Originally formulated for ordered particles, the formulation of the method, proposed by Xiao and Belytschko, remains relevant for amorphous sample used to model geomaterials.

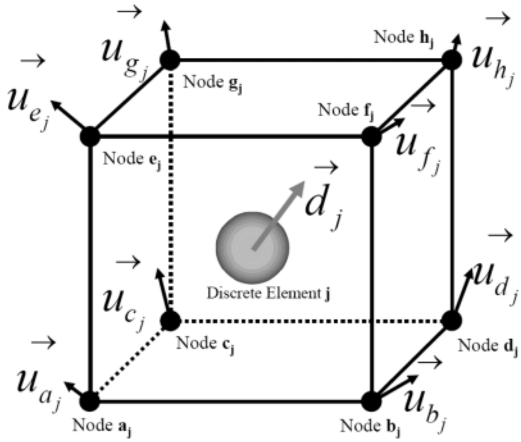

Figure 2. Discrete element position in a continuum volume belonging to bridging domain.

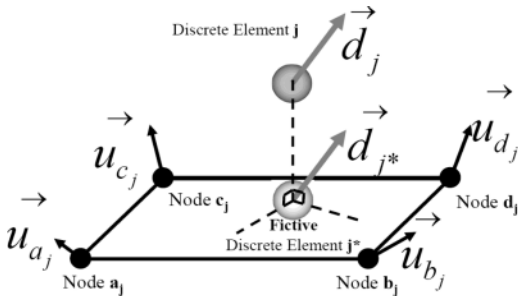

Figure 3. Projection of discrete element position on a continuum surface belonging to "edge-to-edge" domain.

For the simpler "edged-to-edge" model, the Hamiltonian is defined as the sum of discrete and continuum ones (no scaling) because domains are disjoint. Aiming at formulating seamless method for non ordered discrete element sample, the kinematics constraints are calculating using displacement of fictive nodes j^* (localized by position vector X_j^*) obtained by orthogonal projection of position vector X_j in the vicinity of the junction, on the continuum plane border (Fig. 3). Kinematic relations and constrains become (Equation 5 & 6):

$$\overrightarrow{u(\overrightarrow{X_j^*})} = \sum_{i=a}^{d} \overline{\overline{k_{ji}}} \ \overrightarrow{u_i} \tag{5}$$

$$\vec{g} = \overrightarrow{d_{j^*}(\overrightarrow{X_j^*})} - \overrightarrow{u(\overrightarrow{X_j^*})} = \vec{0} \tag{6}$$

where nodes $i = a$ to d, are the 4 nodes surrounding fictive node j^* in the plane junction.

Combined problem is solved by minimizing modified Hamiltonian H_L for the complete model, using the Lagrangian multiplier λ method to ensure continuity in bridging domain (Equation 7). An explicit algorithm is used for dynamical application and the

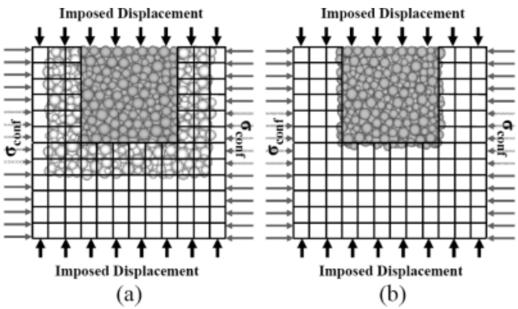

Figure 4. Representation of coupled samples used for static triaxial tests. (a) "bridging" model (b) "edge-to-edge" model.

scheme of resolution is detailed in Xiao Belytschko 2004 and Frangin et al. 2006.

$$H_L = \alpha \; H_{Discrete} + \beta \; H_{Continuum} + \vec{\lambda}.\vec{g} \qquad (7)$$

In our approach, distinct element method (DEM) (PFC^{3D} code, Itasca) is employed in the discrete domain while finite difference method ($FLAC^{3D}$ code, Itasca) is employed in the continuum domain. Implementation of the coupled approach in these two different codes is motivated by the fact they have TCP/IP socket connection ability which permits data transmission at each calculation step to control and correct displacements at combined boundary.

2.2 *Validation of coupling methods*

2.2.1 *Triaxial quasi-static test*

In this section, these coupled methods are compared and validated through static and dynamic tests.

First, a numerical triaxial test is performed on a cubic elastic sample (Fig. 4) (length = 3 m), characterized by a Young's modulus E = 500 MPa and a Poisson coefficient $\upsilon = 0.32$. When it is used, the granular material is modelled by spherical particles of various diameters (ratio of 2.0 between the greater and smaller particles) associated together by rigid elastic bonds. The contact properties in discrete domain are equal to those of "Natural Soil" material in section 3, and are summed up in Table 1 & 2. For quasi-static solicitation, local damping coefficient is set to 0.7 in both discrete and continuum approaches.

The confining pressure σ_{conf} apply on the lateral faces is constant and equal to $\sigma_{conf} = 10$ kPa.

The "bridging" or "edge-to-edge" models can be compared with continuum and discrete approaches. Vertical stresses and lateral strains measured numerically are very close whatever the model employed. Only lateral deformation curve for DE method presents a quite different behaviour. Apparent Young's modulus (Fig. 5) and Poisson coefficient (Fig. 6) are overall unchanged for coupled models: material continuity is ensured at the transition areas. Implementation of 'Edge-to-edge' and 'bridging' coupling methods are validated for elastic material in quasi-static application.

Table 1. Characterization of elasticity in discrete element model.

Elastic's Parameters	k_n N/m	k_s N/m
Gravel	1.77e6 × R	0.35e6 × R
Natural Soil	3.78e6 × R	1.13e6 × R

Table 2. Characterization of plasticity in discrete element model.

Plastic's Parameters	c_f –	b_n N	b_s N
Gravel	1.0	1.78e5 × R^2	1.78e5 × R^2
Natural Soil	∞	∞	∞

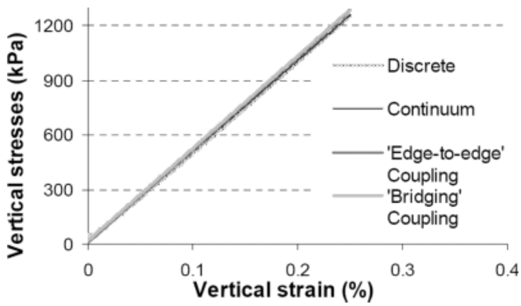

Figure 5. Vertical stresses (Young modulus characterization) in sample during triaxial test: comparison of coupled models with full continuum and full discrete models.

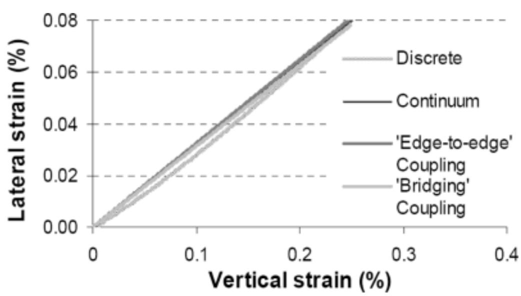

Figure 6. Lateral strain (Poisson's effect) in sample during triaxial test: comparison of coupled models with full continuum and full discrete models.

2.2.2 *Dynamical compression test*

The coupling methods are also tested for dynamical application, in particular for wave propagation into an elastic medium. The same four models are handled for this validation test (Fig. 7). The material properties are identical to those established in the previous static test, except damping coefficient which is set to zero for dynamical application.

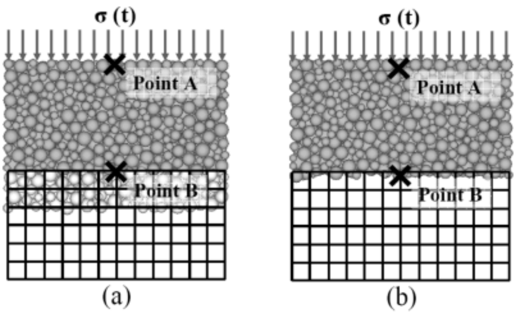

Figure 7. Schematic representation of coupled samples used for dynamic tests. (a) "Bridging" model (b) "Edge-to-edge" model.

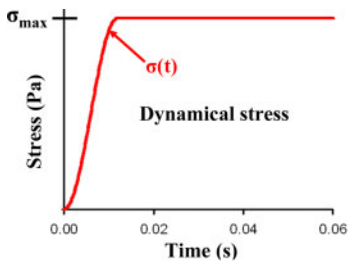

Figure 8. Imposed dynamical stress loading at the top of the discrete element model.

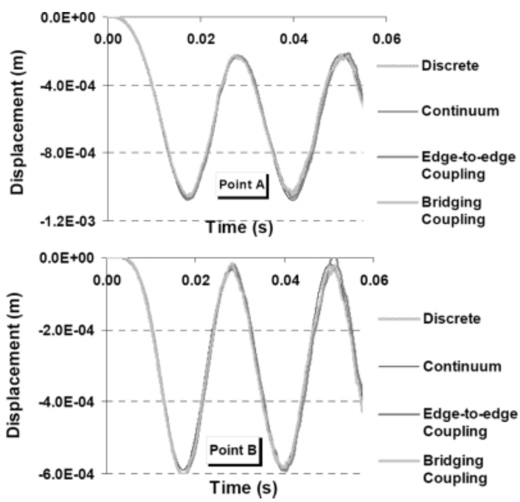

Figure 9. Displacements in samples during dynamical simulation: comparison between different approaches.

A dynamical force load σ(t) is applied on the head of each sample, and in order to evaluate their dynamical behaviour, vertical displacements are recorded at points A(0,0,3) and B(0,0,1.5).

Amplitudes and frequencies numerically calculated are very similar for all samples. It should be noted that errors on peaks don't exceed 3–5% (Fig. 9). These

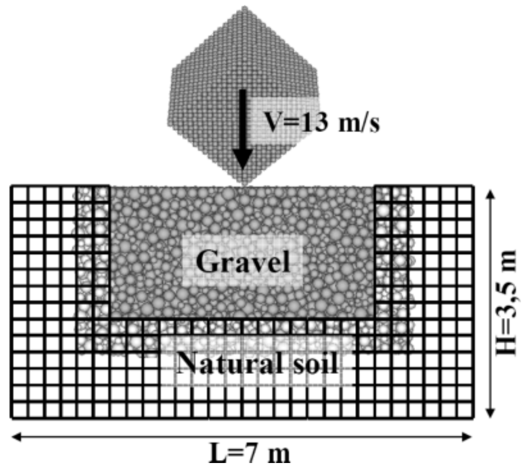

Figure 10. Scheme of the bridging coupled model, defined by two overlapping zones.

insignificant differences are partly inherent to coupled models: constraining degree of liberty can slightly modify local mechanical behaviour.

These previous dynamical tests provide interesting results regarding the capability of combined model to describe dynamical behaviours in elastic domains.

3 IMPACT BLOCK SIMULATION

3.1 *Model description*

In this section we employ coupling methods to investigate modelling possibilities of a rock impact on geomaterial layer or later on a global structure.

The model presented thereafter is adapted from experimental research, described by Pichler et al. 2005, concerning the loading of layers of gravel subjected to rockfall. The experimental approach is based on measurements or estimations of penetration depth, impact force and impact duration when a cubic rock impacts onto the ground with a tip.

The following model is composed by a trench of 4 m width and 2 m depth, dug in natural soil, and filled by well-graded gravel (Fig. 10). In the different approaches, mechanical behaviour of gravel is described by elasto-plastic behaviour to take into account the absorbing energy capacity of gravel. The natural soil is not submitted to high load, and is modelled by elastic behaviour. It can be viewed as a boundary condition.

Bloc impact on gravel is simulated with the "edge-to-edge" and "bridging" combined model, and entirely discrete one in order to validate the numerical processes.

The transition between discrete and continuum approaches must take place in elastic domain (natural soil). Elastic parameters of the gravel cushion were estimated by the mean of tests on the embankment

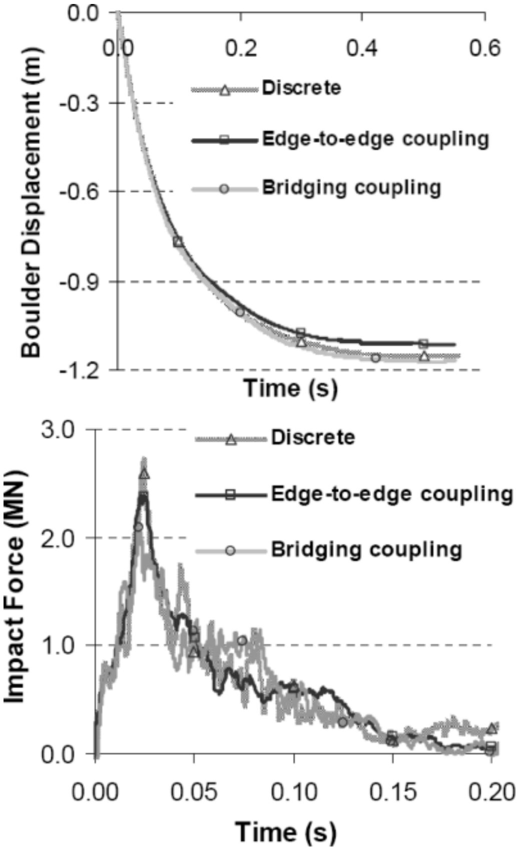

Figure 11. Penetration and impact force calculated by coupled and discrete approaches.

($E = 196$ MPa, $\upsilon = 0.36$), but no significant information were available to estimate plasticity's parameters.

First, to validate the combined discrete – continuum approach the discrete element model retained is rather simple, composed only by spherical element. The inter-particular interactions are characterized by local normal and shear stiffness's (k_n and k_s), normal and shear local bonds (b_n and b_s), and a local friction coefficient (c_f). Using spherical elements tends to limit the macro friction angle to values around 30° (Chareyre 2003) which can be considered as a low value for gravel. The choice of a local friction coefficient $c_f = 1.0$ (45°) lead to a macro friction angle of $\phi = 29°$, and the local bond normal b_n and shear b_s strength are fitted to obtained a macro cohesion $c = 16$ kPa. Micro-mechanical parameters (k_n and k_s) of the natural soil insure an elastic behaviour characterized by $E = 500$ MPa and $\upsilon = 0.32$).

The mechanical parameters for discrete numerical simulation are summarized Table 1 & 2. It should be noted that values of k_n, k_s, b_n and b_s are not intrinsic to a material but depend on particle radius chosen for soil modelling. Note that damping coefficient is kept to zero for impact simulation.

Table 3. Penetration and impact force obtained experimentally and simulated numerically for an 850 kJ impact.

Block $V = 13$ m/s & $m = 10^4$ kg	Experimental Pichler et al. (2005)	Numerical
Penetration	0.51 m	1.15 m
Max. Impact Force	3.5 MN	2.5 MN

The shape of the impactant block is quite cubic, has a masse of around $m = 10{,}000$ kg, and is supposed very stiffer compared to the impacted soil. Fall is not calculated but consequently, the velocity of the boulder was initialized to $v = 13$ m/s, which correspond to a 8.5 m high fall. The kinetic energy of the impact is almost $E = 850$ kJ which corresponds to the experimental work of Pichler et al. (2005).

3.2 *Calculation results and comments*

During impact, penetration and resulting force on the cubic impactant are recorded to be compared to experimental measures (Pichler et al. 2005).

The response of the model based on coupling methods is close to the entirely discrete one (Fig. 11), and draws the prospects of this approach to model large geometry of structure which need locally accurate description.

However, due to simplifying assumptions made to model the gravel soil, the comparison with experimental and analytical results given by Pichler et al. (2005) show that dynamical resistance of the gravel layer is under-estimated. Indeed, experimental penetration and analytical impact force are respectively 50% lower and 40% upper (Table 3).

These differences can be partially explained by both static characterisation of the mechanical parameters and no consideration of dynamical effect during impact (energy dissipation, local plasticity, braking of grains, etc.).

First, the macro mechanical parameters ($\varphi = 29°$, $c = 16$ kPa), affected to the gravel material to describe plasticity, are determinate by the analysis of the peak resistance for triaxial. Post-failure, resistance is very weaker, mainly because of the low residual resistance retained for well-graded gravel in large deformations. Thus, mechanical behaviour needs to be updated to simulate upper friction angle in geomaterial, in introducing rolling resistance at each contact (Plassiard 2007), in inhibiting rotation degrees of freedom (Calvetti et al. 2005) or using non-spherical particles (Bertrand et al. 2005, Salot et al. 2009) for instance.

Another explanation is issued from the phenomenon of particles ejection observed numerically around impact (Fig. 12). The model seems not to dissipate enough energy, and the elastic restitution is over-estimated, for shallow discrete element, compared to experimental observations. The actual constitutive law, calibrated from quasi-static triaxial tests, can not

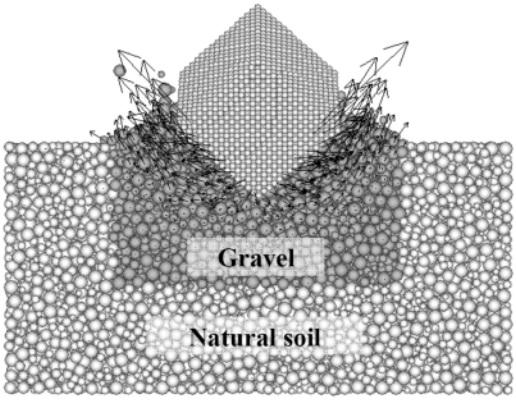

Figure 12. Cross section of the DE model after impact. Visualisation of particles ejection in the vicinity of impact area.

model phenomenon as viscosity, compaction and local breaking in high loaded areas.

Dissipations, due to interparticle sliding (friction) or break of cohesive bond, are not sufficient to represent all sources of dissipation during impact. Aiming at improving the response of the granular material, and in the same time the validity of this model, additional dissipative laws need to be implemented.

4 CONCLUSION

An innovative combined discrete – continuum element method has been adapted to study mechanical behaviour of large civil and geotechnical engineering structures. Validated by means of static and dynamical elementary tests, prospects of coupling methods are then evaluated through the boulder impact on a gravel layer. In this case, the good accordance between discrete and continuum – discrete element methods shows the interest of such method to describe locally impact phenomenon.

At this step, the limits of the numerical model are based on the description of the dissipative behaviour of high loading granular material. Local constitutive laws needs to be further developed to take into account dynamical behaviour of granular under high energy impact.

In the framework of REMPARe research project (www.rempare.fr), high energy impact experiments (2000 kJ) have been performed to test protection dams at real scale. Experimental data should permit us to feat our model in order to optimize and improve protection structure design.

ACKNOWLEDGEMENTS

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